# EXERGY LOSS ON MIXING WORKING BODIES WITH DIFFERENT TEMPERATURES

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A thermodynamic analysis is presented for the mixing of working bodies differing in temperature.

The mixing of working bodies is widely used in technological schemes. If the bodies differ in temperature, pressure, composition, or concentration, the mixing is accompanied by exergy loss, which is made up of contributions from each of the parameters. If the bodies differ only in temperature, we have a particular case, but one which occurs widely in industrial temperature-control systems. The exergy loss is determined only by the temperature difference. Determination of this is not only of theoretical significance, but also of practical value, since such losses affect the working parameters (inadequate degree of cooling in a coolant, increase in the necessary heat-transfer surface, etc.) [1].

The absolute specific exergy losses have been determined [2] on mixing bodies differing in temperature. The present study is a continuation of [2] and deals with the thermodynamic analysis of such processes with the use of the exergy efficiency and the exergy loss coefficient. It is assumed that the processes occur adiabatically and that the specific heats of the working bodies are constant in the working temperature range, and also that there is complete mixing of the flows and only the thermal component of the exergy need be considered.

The following is [3, 4] the specific thermal exergy of a working body:

$$e = c\left[T - T_0 - T_0 \ln\left(\frac{T}{T_0}\right)\right]. \tag{1}$$

The exergetic efficiency in mixing may be written as

$$\eta_e = 1 - \frac{\text{loss exergy}}{\text{exergy broughtin}} = 1 - \frac{\nabla e_m}{e_1 + e_2}. \tag{2}$$

The denominator in the second term in (2) can be represented as the sum of the specific thermal exergy of the flow mixture, which is defined by (1) for the mixture temperature $T_m = T_2 + (T_2 - T_1)K$ and the absolute specific exergy losses in the process

$$e_1 + e_2 = e_m + \nabla e_m. \tag{3}$$

The following are [2] the absolute specific exergy losses in the process:

$$\nabla e_m = cT_0\left[\ln\left(1 - K + \frac{T_1}{T_2}K\right) - K \ln\left(\frac{T_1}{T_2}\right)\right]. \tag{4}$$

On the basis of (1)-(4), the exergetic efficiency of the mixing may be put as

$$\eta_e = 1 - \frac{T_0 c\left[\ln\left(1 - K + K\frac{T_1}{T_2}\right) - \right. \rightarrow \ldots}{c\left[T_2 + (T_1 - T_2)K - T_0 - T_0 \ln\left(\frac{T_2 + T_1 K - T_2 K}{T_0}\right)\right] + }$$

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$$\cdots \rightarrow \frac{-K\ln\left(\frac{T_1}{T_2}\right)\Big]}{+T_0c\left[\ln\left(1-K+K\frac{T_1}{T_2}\right)-K\ln\left(\frac{T_1}{T_2}\right)\right]}. \tag{5}$$

The corresponding transformations give

$$\eta_e = 1-\frac{\ln\left(1-K+K\frac{T_1}{T_2}\right)-K\ln\left(\frac{T_1}{T_2}\right)}{\frac{T_2}{T_0}\left(1-K+K\frac{T_1}{T_2}\right)-1-\ln\left(\frac{T_2}{T_0}\right)-K\ln\left(\frac{T_1}{T_2}\right)}. \tag{6}$$

To determine the effects from the ratio of the absolute temperatures $T_1/T_2$ on the exergetic efficiency, we consider (2), which can be put as follows on the basis of (3):

$$\eta_e = 1-\left(\frac{e_m}{\nabla e_m}+1\right)^{-1}. \tag{7}$$

If we increase $T_1/T_2$ in (7), which determines $e_m$ and $\nabla e_m$, the specific exergy of the mixture will remain unchanged, while the absolute losses will [2] increase. Consequently, the exergetic efficiency will decrease as $T_1/T_2$ rises.

To determine the effects on the exergetic efficiency from the environmental temperature $T_0$ or the dimensionless ratio $T_2/T_0$, we differentiate (6) with respect to $T_2/T_0$ and equate the result to zero:

$$\frac{\partial\eta_e}{\partial\left(\frac{T_2}{T_0}\right)} = -\frac{\left[\ln\left(1-K+K\frac{T_1}{T_2}\right)-K-\ln\left(\frac{T_1}{T_2}\right)\right]\left[1-K+K\frac{T_1}{T_2}-\left(\frac{T_1}{T_2}\right)^{-1}\right]}{\left[\frac{T_2}{T_0}\left(1-K+K\frac{T_1}{T_2}\right)-1-\ln\left(\frac{T_2}{T_0}\right)-K\ln\left(\frac{T_1}{T_2}\right)\right]^2} = 0, \tag{8}$$

whence the values of $(T_2/T_0)_{min}$ for which the exergetic efficiency attains its minimum values are

$$\left(\frac{T_2}{T_0}\right)_{min} = \left(1-K+K\frac{T_1}{T_2}\right)^{-1}. \tag{9}$$

When the bodies have different signs for the exergy, and the temperature of one body is above the environmental value, while the other is below it ($T_1 > T_0 > T_2$), then $(T_2/T_0)_{min}$ lies in the range $1 > T_2/T_0 > (T_1/T_2)^{-1}$. The mixing coefficient in (9) varies in the range 1-0, so $(T_2/T_0)_{min}$ will also be in the range $1-(T_1/T_2)^{-1}$; consequently, on mixing flows with different exergy signs, $\eta_e$ will attain its minimum value. In the limit, $\eta_e$ may be zero when $T_m = T_0$ as a result of the mixing. If the flows have identical signs for the exergy and the temperature is positive ($T_2 > T_0$), $\eta_e$ will decrease as $T_0$ rises, while it increases in the negative-temperature zone. In other words, the closer the temperatures of the mixing bodies to $T_0$, the less will be $\eta_e$ under otherwise equal conditions.

To determine $K_{min}$, for which $\eta_e$ attains its minimum value, we differentiate (6) with respect to K and equate the result to zero:

$$\begin{aligned}\frac{\partial\eta_e}{\partial K} = &-\left[\frac{T_2}{T_0}\left(\frac{T_1}{T_2}-1\right)-\ln\left(\frac{T_1}{T_2}\right)\right]\left[\ln\left(1-K+K\frac{T_1}{T_2}\right)-\right.\\ &\left.-K\ln\left(\frac{T_1}{T_2}\right)\right]+\left[\left(\frac{T_1}{T_2}-1\right)\left(1-K+K\frac{T_1}{T_2}\right)^{-1}-\ln\left(\frac{T_1}{T_2}\right)\right]\times\\ &\times\left[\frac{T_2}{T_0}\left[1-K+K\frac{T_1}{T_2}\right)-1-\ln\left(\frac{T_2}{T_0}\right)-K\ln\left(\frac{T_1}{T_2}\right)\right] = 0.\end{aligned} \tag{10}$$

Figure 1 shows the numerical solution to (10), which indicates that $K_{min}$ is less than 0.5 in the positive-temperature range, but it is more than 0.5 for negative temperatures. As $T_1/T_2$ increases in the positive range of temperatures, $K_{min}$ decreases, while it increases in the

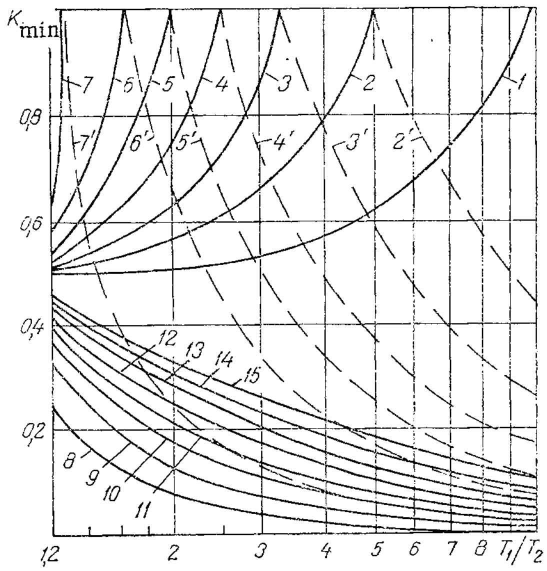


Fig. 1. Dependence of $K_{min}$ on $T_1/T_2$ and $T_2/T_0$: 1-7) $T_0 > T_1 > T_2$; 2'-7') $T_1 > T_0 > T_2$; 8-15) $T_1 > T_2 > T_0$; 1) $T_2/T_0 = 0.1$; 2, 2') 0.2; 3, 3') 0.3; 4, 4') 0.4; 5, 5') 0.5; 6, 6') 0.6; 7, 7') 0.8; 8) 1.1; 9) 1.2; 10) 1.4; 11) 1.6; 12) 2; 13) 3; 14) 5; 15) 10; T, °K.

negative range. As $T_1/T_2$ decreases, $K_{min} \to 0.5$. The figure also shows curves for determining $K_{min}$ on mixing bodies with different exergy signs. For these values of $K_{min}$, $T_m = T_0$ and $\eta_e = 0$, as stated above. The curves are described by the equation

$$K_{min} = \left(\frac{T_0}{T_2} - 1\right) \Big/ \left(\frac{T_1}{T_2} - 1\right). \tag{11}$$

One sometimes also uses the exergy loss coefficient, which is defined as the ratio of the lost exergy to that brought in and is equal to $r = 1 - \eta_e$; therefore, the dependence on $T_1/T_2$ and $T_2/T_0$ is analogous to that for the exergetic efficiency, but the sense is the opposite. For this reason, the dependence of the exergy loss coefficient on the mixing coefficient is also of turning-point type and one gets the maximum value of $r_{max}$ at the point at which the exergetic efficiency attains its minimum value.

These relationships enable one to perform a thermodynamic analysis for any system in which working bodies with different temperatures mix.

## NOTATION

T, temperature; e, specific energy; c, specific heat; $\nabla e$, absolute specific exergy losses; $\eta_e$, exergetic efficiency; r, exergy loss coefficient; W, test body flow; K, mixing coefficient. Subscripts: 1, test body with higher temperature; 2, test body with lower temperature; m, mixture; 0, ambient medium.

## LITERATURE CITED

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